Solving Problems Using Monte Carlo Simulation

OSMAN BALCI
Professor

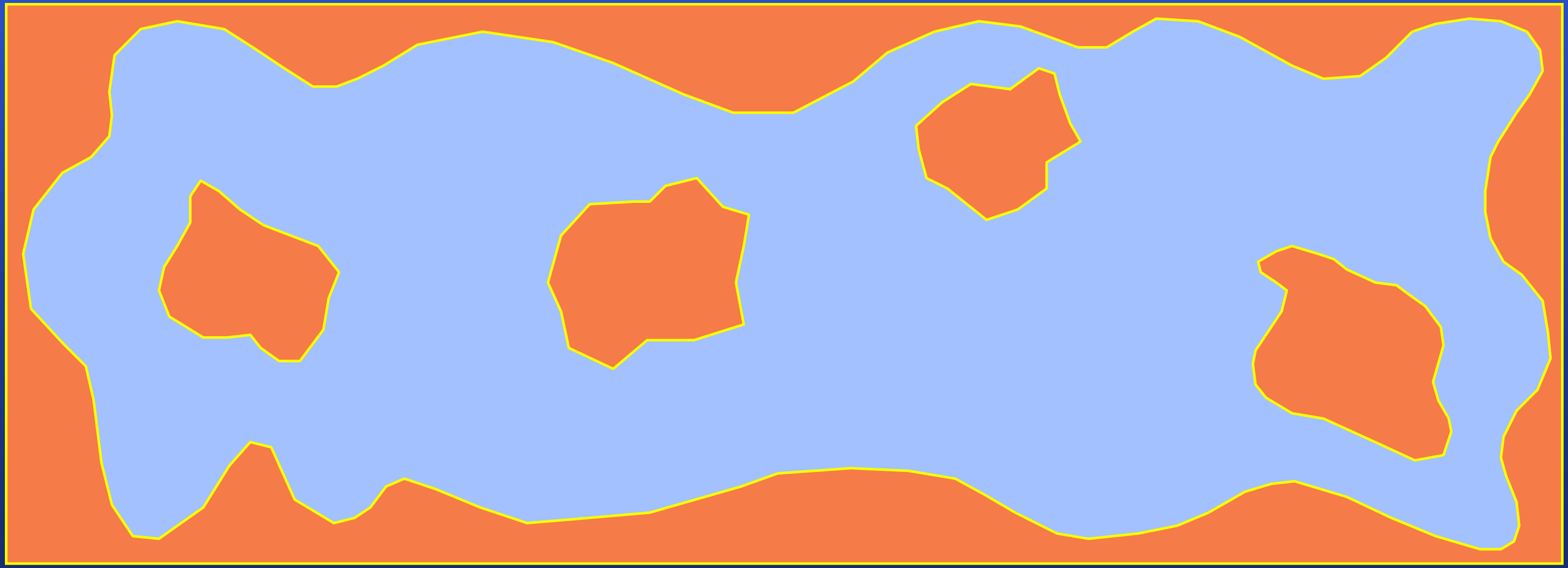
Department of Computer Science
Virginia Polytechnic Institute and State University (Virginia Tech)
Blacksburg, VA 24061, USA

https://manta.cs.vt.edu/balci

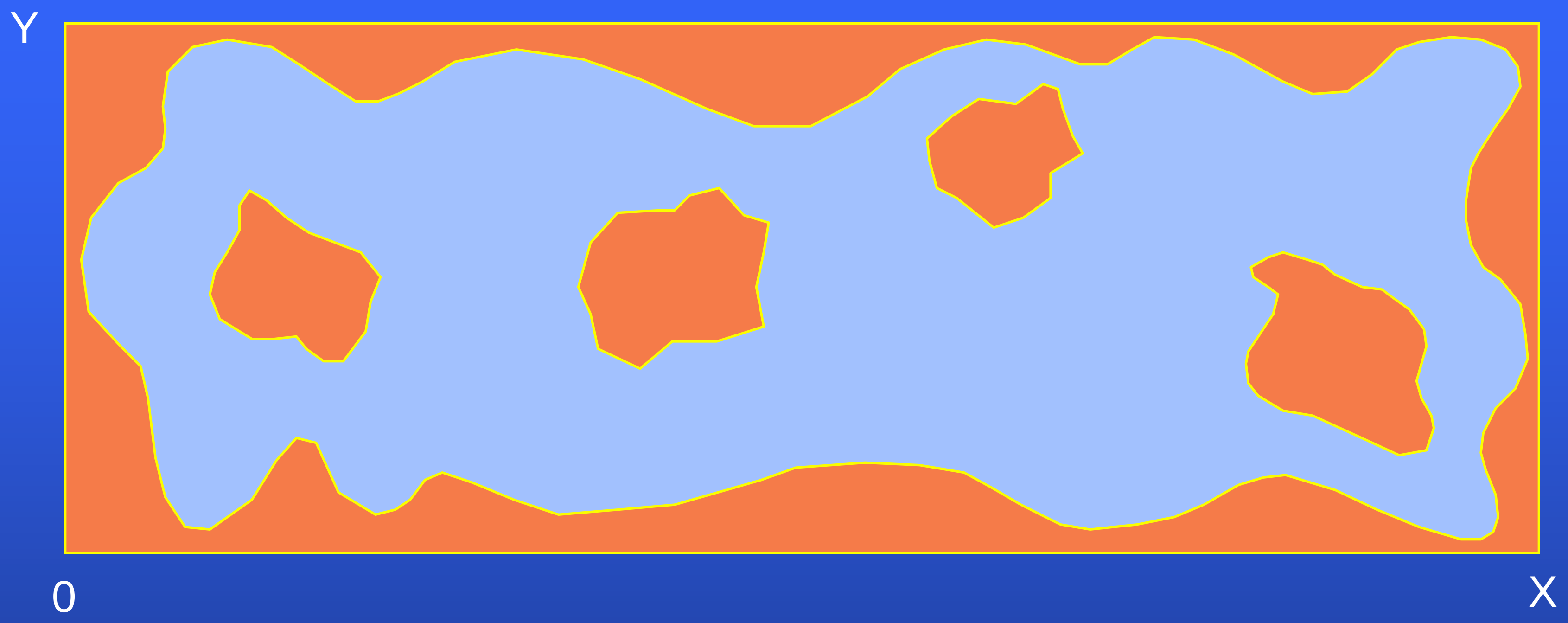
Problem

- Given a high-resolution computer image of a map of an irregularly shaped lake with several islands, determine the water surface area. Assume that the x-y coordinates of every point on the map can be measured.

  Suggest alternative solution approaches!



Monte Carlo Simulation



Step 1:

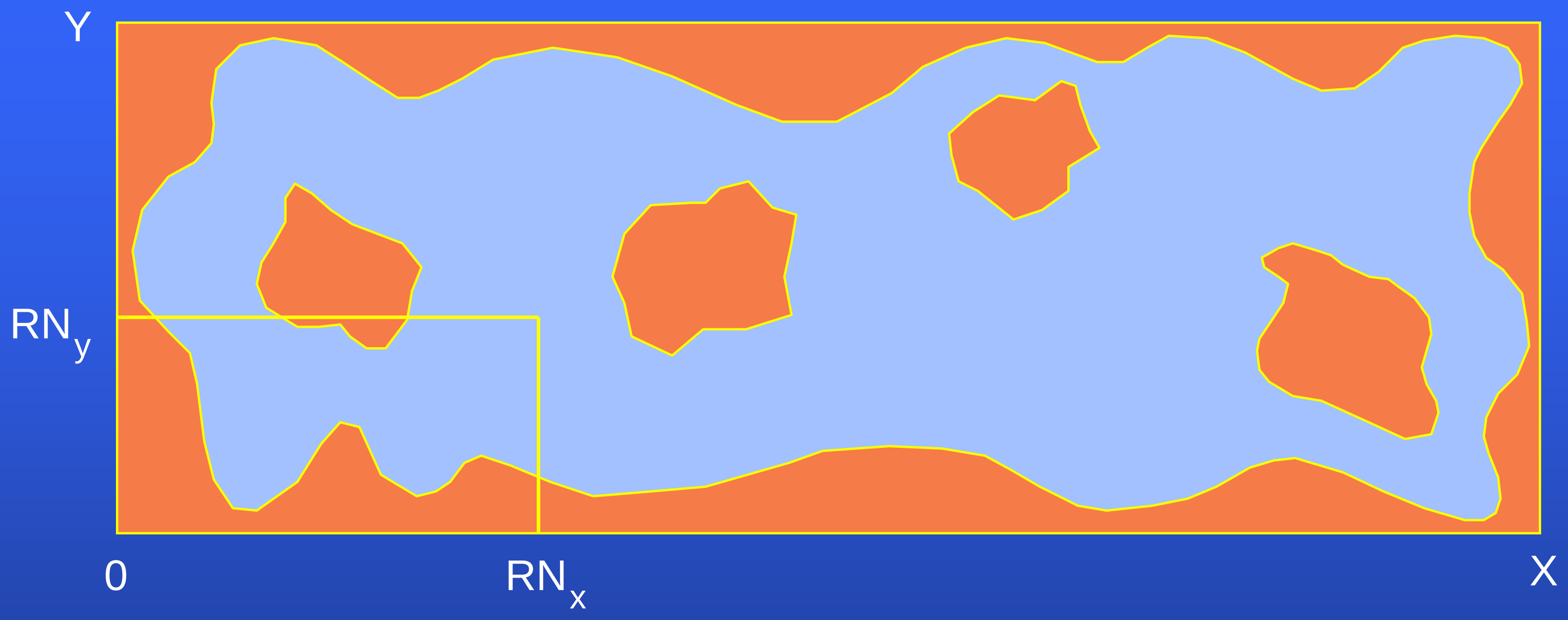
Enclose the area of interest in the smallest rectangle of known dimensions X and Y. Set j = 1, S = 0, and choose a large value for N where:

j = trial number

S = number of hits on the water surface area

N = total number of trials

Monte Carlo Simulation



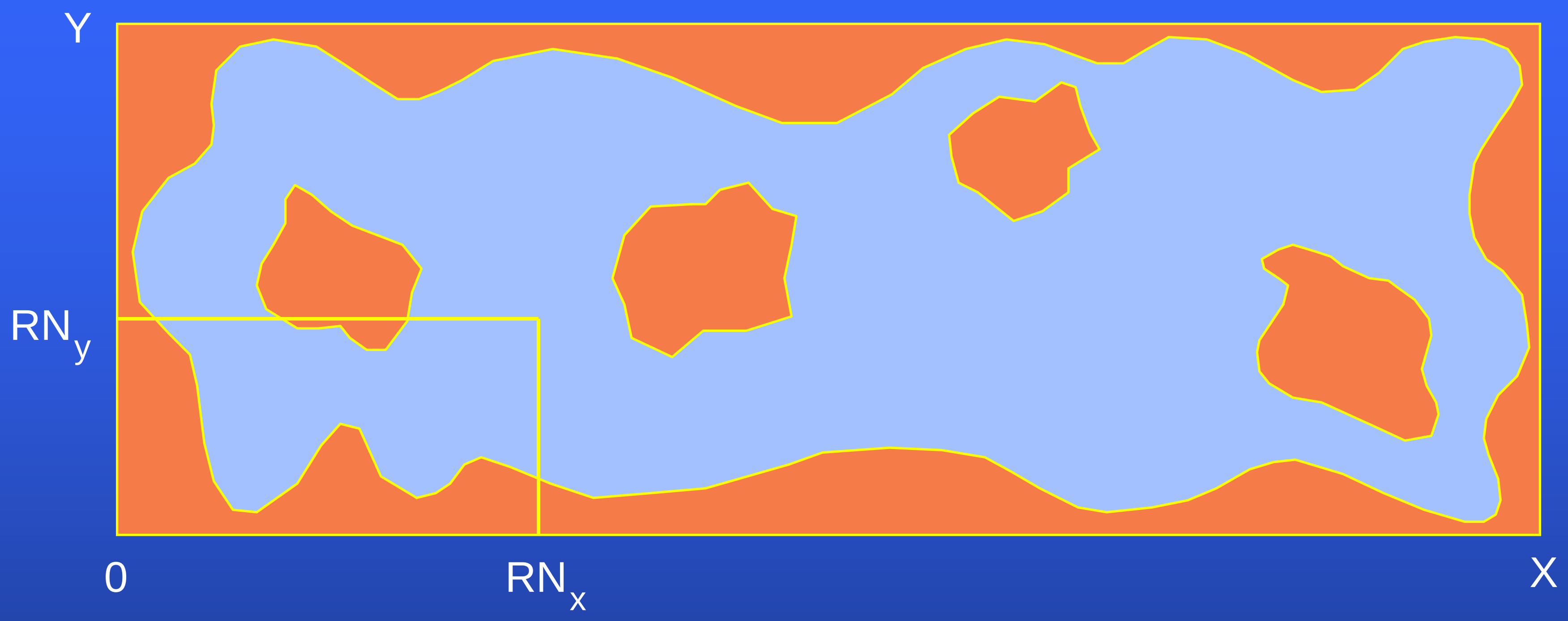
Step 2:

Generate a uniformly distributed random number, RN_x over the length of X.

Step 3:

Generate another uniformly distributed random number, RN_y over the length of Y.

Monte Carlo Simulation



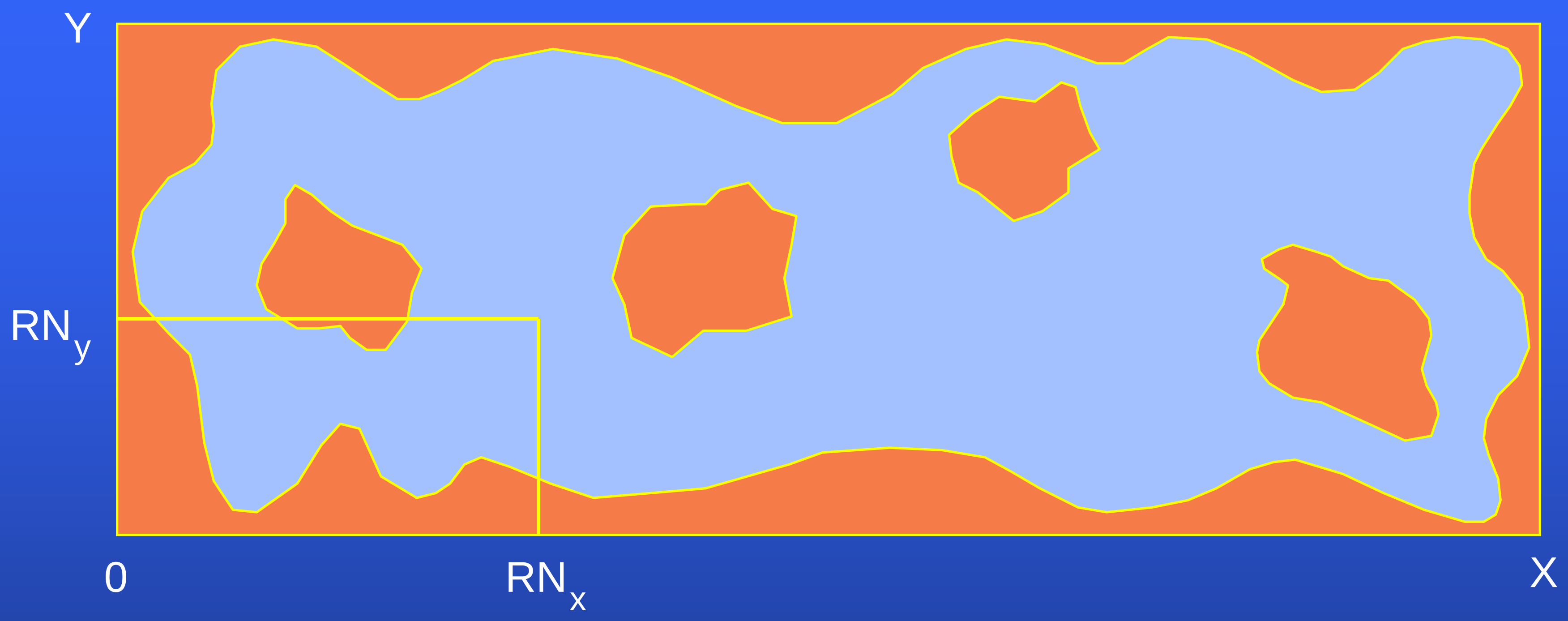
Step 4:

If the point of intersection, (RN_x, RN_y), falls on the water surface area, add 1 to S.

Step 5:

Add 1 to j. If j > N, go to Step 6; otherwise, go to Step 2.

Monte Carlo Simulation



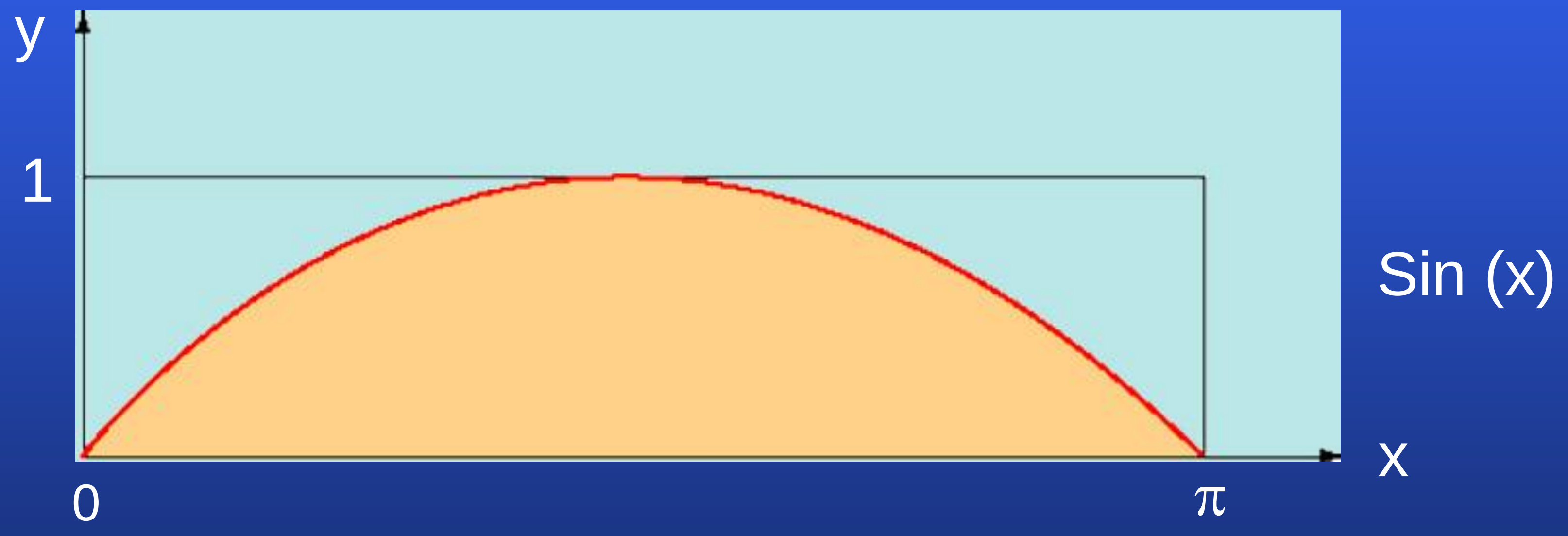
Step 6:

The estimate of the water surface area, $\bar{A}$, is given by: $\frac{\bar{A}}{X\,Y} = \frac{S}{N}$

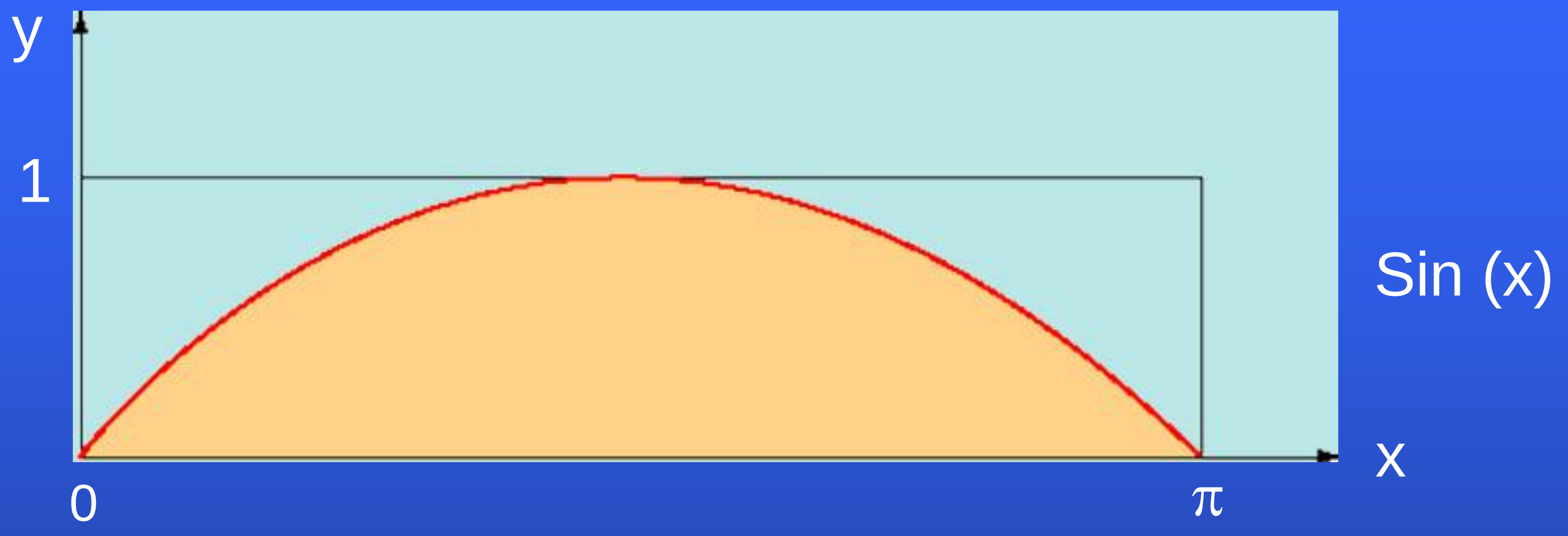
NOTE: As $N \rightarrow \infty$, $\bar{A} \rightarrow$ true value of the area

Problem

- **Use the fundamental theory and logic of the Monte Carlo Simulation technique to estimate the area under the Sine curve over 0 and π.**



Monte Carlo Simulation

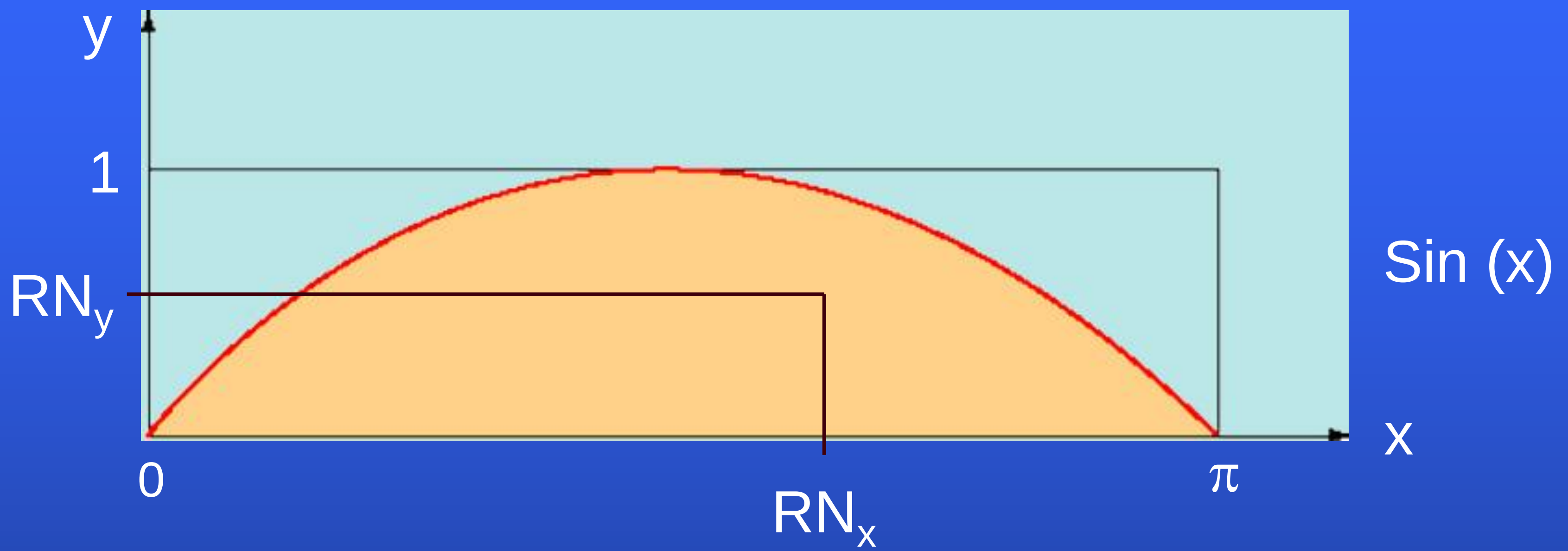


Step 1:

Enclose the area of interest in the smallest rectangle of known dimensions π and 1. Set j = 1, S = 0, and choose a large value for N where:

- **j = trial number**
- **S = number of hits on the area under the Sin (x) curve**
- **N = total number of trials**

Monte Carlo Simulation



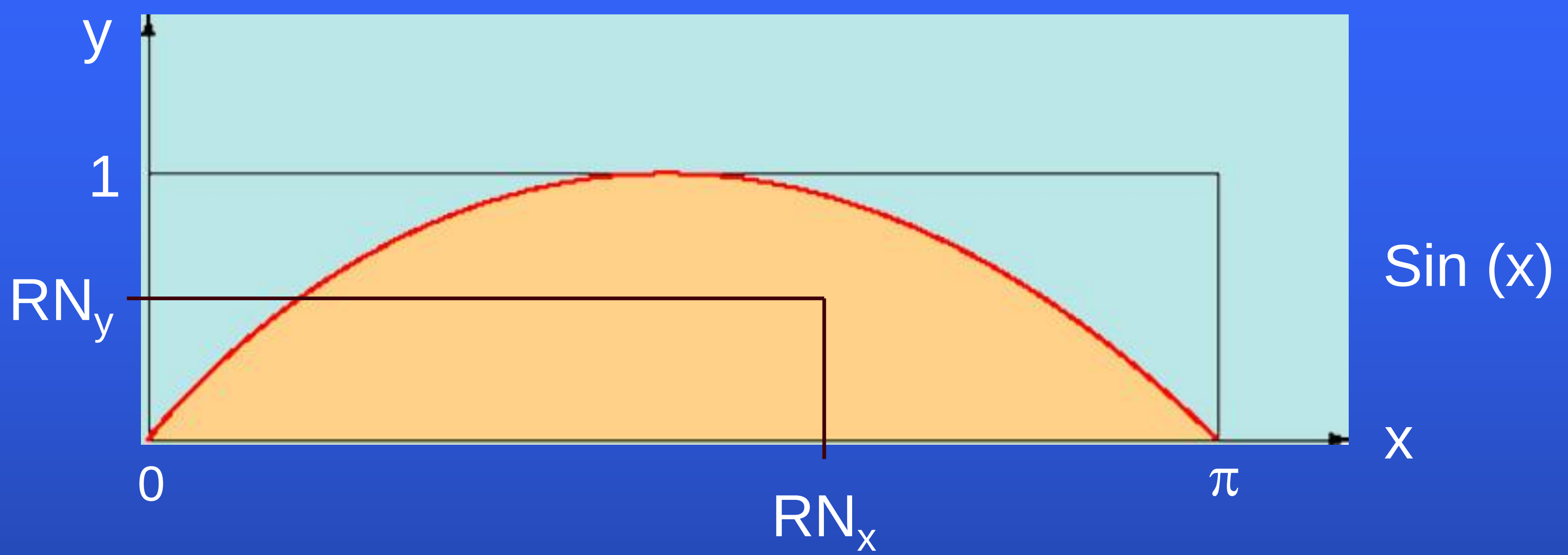
Step 2:

Generate a uniformly distributed random number, RN_x over the length of π.

Step 3:

Generate another uniformly distributed random number, RN_y over the length of 1.

Monte Carlo Simulation



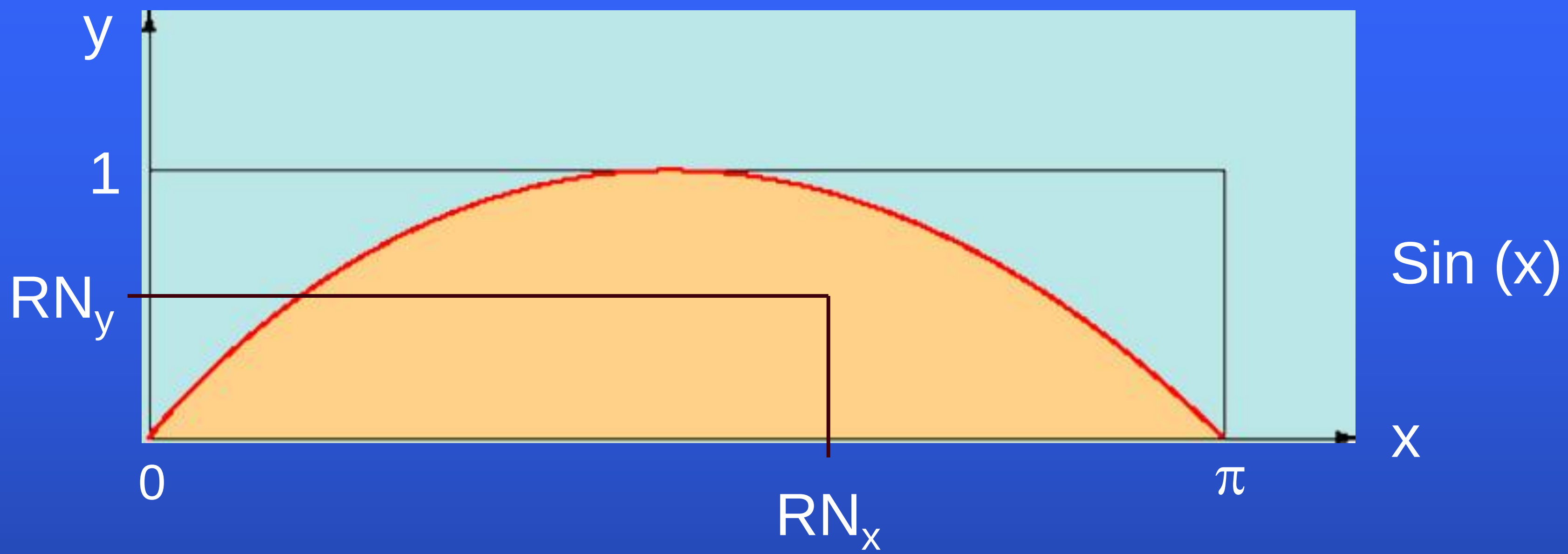
Step 4:

If the point of intersection, (RN_x, RN_y), falls on or below the Sin (x) curve (i.e., if $RN_y \leq \sin(RN_x)$), add 1 to S.

Step 5:

Add 1 to j. If j > N, go to Step 6; otherwise, go to Step 2.

Monte Carlo Simulation



Step 6:

The estimate of the area, $\bar{A}$, is given by: $\frac{\bar{A}}{\pi} = \frac{S}{N}$

NOTE: **As $N \to \infty$, $\bar{A} \to 2$ (true value of the area)**

Problem

- Use the fundamental theory and logic of the Monte Carlo Simulation technique to solve the following optimization problem:

Maximize $Z = (e^{X_1} + X_2)^2 + 3 (1 - X_3)^2$

Subject to:

$0 \leq X_1 \leq 1$

$0 \leq X_2 \leq 2$

$2 \leq X_3 \leq 3$

Monte Carlo Simulation

- Use the fundamental theory and logic of the Monte Carlo Simulation technique to solve the following optimization problem:

  Maximize $Z = (e^{X_1} + X_2)^2 + 3 (1 - X_3)^2$

  Subject to:

  $0 \leq X_1 \leq 1$

  $0 \leq X_2 \leq 2$

  $2 \leq X_3 \leq 3$

- **Step 1:**

  Set j = 1 and choose a large value for N where:

  j = trial number

  N = total number of trials

Monte Carlo Simulation

- Use the fundamental theory and logic of the Monte Carlo Simulation technique to solve the following optimization problem:

Maximize $Z = (e^{X_1} + X_2)^2 + 3 (1 - X_3)^2$

Subject to:

$0 \leq X_1 \leq 1$

$0 \leq X_2 \leq 2$

$2 \leq X_3 \leq 3$

- **Step 2:**
  Generate a proper random number, RN_1, over 0 and 1.
- **Step 3:**
  Generate another proper random number, RN_2, over 0 and 2.
- **Step 4:**
  Generate another proper random number, RN_3, over 2 and 3.

Monte Carlo Simulation

- Use the fundamental theory and logic of the Monte Carlo Simulation technique to solve the following optimization problem:

Maximize $Z = (e^{X_1} + X_2)^2 + 3 (1 - X_3)^2$

Subject to:

$0 \le X_1 \le 1$

$0 \le X_2 \le 2$

$2 \le X_3 \le 3$

- **Step 5:**
  Substitute RN_1 for X_1 , RN_2 for X_2 , and RN_3 for X_3 in the objective function. Store its value in Z(j) and record the corresponding values for X_1 , X_2 , and X_3.
- **Step 6:**
  Add 1 to j. If j > N, go to Step 7; otherwise, go to Step 2.

Monte Carlo Simulation

- Use the fundamental theory and logic of the Monte Carlo Simulation technique to solve the following optimization problem:

  Maximize $Z = (e^{X_1} + X_2)^2 + 3 (1 - X_3)^2$

  Subject to:

  $0 \leq X_1 \leq 1$

  $0 \leq X_2 \leq 2$

  $2 \leq X_3 \leq 3$

- **Step 7:**
  The approximate solution of the problem is determined by the values of X_1 (= RN_1), X_2 (= RN_2), and X_3 (= RN_3), which correspond to the maximum value of { Z(j), j = 1, 2, 3, ..., N }.

- NOTE: As $N \to \infty$, $X_1 \to 1$, $X_2 \to 2$, and $X_3 \to 3$.